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Aggregational Gaussianity and barely infinite variance in financial returns

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ABSTRACT

This paper aims at reconciling two apparently contradictory empirical regularities of financial returns, namely, the fact that the empirical distribution of returns tends to normality as the frequency of observation decreases (aggregational Gaussianity) combined with the fact that the conditional variance of high frequency returns seems to have a (fractional) unit root, in which case the unconditional variance is infinite. We provide evidence that aggregational Gaussianity and infinite variance can coexist, provided that all the moments of the unconditional distribution whose order is less than two exist. The latter characterizes the case of Integrated and Fractionally Integrated GARCH processes. Finally, we discuss testing for aggregational Gaussianity under barely infinite variance. Our empirical motivation derives from commodity prices and stock indices, while our results are relevant for financial returns in general.

1. Introduction

One of the most important questions in the financial literature concerns the distribution of financial prices. Kendall (1953) was the first to notice that the empirical distributions of successive price changes deviate from normality mainly because they exhibit excess kurtosis. Mandelbrot (1963) suggested that the observed leptokurtosis reflects that the variance of commodity or stock price changes is infinite. Specifically, Mandelbrot observed that the logarithmic price changes (hereafter referred to as 'returns') within a specific period of time, is the sum of elementary returns, ξ_i, between transactions that occur during this period. He then assumed that the variance of returns is infinite, which in turn implies that the Central Limit Theorem is not applicable. As a result, the sum of ξ_i's converges not to the normal distribution, but instead, to a Stable Paretian distribution. The latter is leptokurtic and has infinite variance. An alternative explanation for the observed leptokurtosis in the empirical distributions of returns was offered by, among others, Clark (1973) and Blattberg and Gonedes (1974). These studies argue that transactions are not spread uniformly across time, which in turn implies that the underlying distribution of returns is a mixture of normals, therefore the finite-variance assumption is not sacrificed.

The two competing explanations for leptokurtosis mentioned above, bare different implications about the behavior of the distribution of returns as we move from higher (say daily) to lower (say monthly) frequencies of observations. In particular, as we move from higher to lower frequencies, the degree of leptokurtosis diminishes and the empirical distributions tend to approximate normality. This stylized fact, referred to as "Aggregational Gaussianity", can be accounted for only by the 'mixture of

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normals' explanation of leptokurtosis and not by the infinite-variance alternative. Indeed, if the daily returns follow a stable Paretian distribution with characteristic exponent equal to $a<2$, then, under general dependence assumptions, all the moments of order $s\geq a$ of the monthly (or annual) returns will be infinite.[1] Hence, the property of infinite variance, implied by the stable-Paretian hypothesis, cannot coincide with aggregational Gaussianity (see also Campbell et al., 1997, p. 19).

In late 1980s, when GARCH models emerged, the issue of the parallel existence of infinite variance and aggregational Gaussianity re-emerged. The estimation of GARCH models for commodity or stock returns seemed to suggest: (i) the presence of a unit root (or long memory) in the conditional variance, and (ii) the gradual declining of conditional heteroskedasticity and the associated leptokurtosis of the unconditional distribution as we move from higher to lower frequencies of observation (see Diebold, 1988; Drost and Nijman, 1993). The general class of ARCH (∞) models was introduced by Robinson (1991) and includes as special cases several well known conditionally heteroskedastic models, as the ARCH and GARCH. Baillie et al. (1996), introduced the Fractionally Integrated GARCH (FIGARCH) models which, including IGARCH, are also special cases of ARCH (∞) models (see Giraitis et al., 2009, for a discussion).[2]

Nelson (1990) proved the existence of a stationary solution of the IGARCH (1,1) model. However, the derivation of sufficient conditions for the strict stationarity of the general FIGARCH model was proved to be more difficult. For example, Kazakevičius and Leipus (2003) proved the existence of a strictly stationary solution for the Integrated ARCH (∞) model under a condition that excludes the case of FIGARCH, and Zaffaroni (2004) established necessary and sufficient conditions for the second order stationarity of ARCH (∞) models. Only recently, Douc et al. (2008) introduced a sufficient condition for the stationarity of several classes of ARCH (∞) models, and provided an example where this condition is satisfied by a FIGARCH process.

Overall, empirical studies seem to suggest the simultaneous presence of two seemingly contradictory facts: aggregational Gaussianity and infinite variance. This inconsistency between infinite variance and aggregational Gaussianity motivated some authors to argue that the evidence of long memory in the conditional variance was in fact spurious. It may arise either from structural breaks in the unconditional variance (see Diebold, 1986; Diebold and Lopez, 1995; Lamoureux and Lastrapes, 1990) or from regime switching of the parameters of the conditional variance (see Fong and See, 2001; Fornari and Mele, 1997, among others). Even in these cases, however, an infinite variance stationary model may arise. For example, Liu (2009) introduces the Integrated Markov Switching GARCH model, for which he proves stationarity and infinite variance. Liu's result implies that infinite variance can occur even if the conditional variance parameters in all-but-one regimes correspond to a finite variance model.

In this paper we aim at reconciling aggregational Gaussianity and infinite variance without bringing in question the FIGARCH specification. We show that coexistence of infinite variance and aggregational Gaussianity is possible, provided that all the moments of the unconditional distribution whose order is less than two, exist. This moment condition is satisfied in certain classes of ARCH (∞) processes, including FIGARCH, implying that a FIGARCH process is indeed a process with barely infinite variance (see Douc et al., 2008; Kourogenis and Pittis, 2008).

The paper is organized as follows: In Section 2 we present evidence indicating that the returns of three stock indices and nine commodities observed at high frequencies exhibit both leptokurtosis and long memory in the conditional variance. We also show that these effects tend to diminish as we move to lower frequencies. In Section 3 we explain why there is no paradox in admitting the simultaneous existence of aggregational Gaussianity and FIGARCH and discuss whether the mixing properties of a FIGARCH process conform to those assumed in the relevant limit theorems. In Section 4 we discuss testing for aggregational Gaussianity under infinite variance and present some additional empirical evidence supporting the coexistence of infinite variance and aggregational Gaussianity. The last section concludes the paper.

2. Empirical motivation: distributional characteristics of returns

The motivation for this paper derives from the changes observed in the distributional characteristics of financial returns, as we move from higher to lower frequencies. To empirically illustrate these changes, we use a dataset on spot prices of nine commodities obtained from S&P Goldman Sachs Commodity Indices (corn, soybeans, wheat, gold, silver, cattle, hogs, gasoline and copper) and three major stock indices (S&P 500, DAX 30 and NIKKEI 225). As in Baillie et al. (2007), we filter out the deterministic periodicities observed in the daily corn and soybean returns volatility by applying the Fourier Flexible Form (Gallant, 1981). As illustrated in the supplemental file of the paper (Fig. S1), all distributions are leptokurtic for the daily frequency, while the degree of leptokurtosis of the empirical distributions decreases as we move from daily to annual returns. Table 1 presents the median of the kurtosis coefficient, and the Jarque–Bera and Anderson–Darling test statistics for all returns under consideration and for all examined frequencies.[3] The evolution of the kurtosis coefficient suggests that when the frequency of the observations decreases distributions become less leptokurtic, while Jarque–Bera and Anderson–Darling statistics do not reject the null of normal distributions in low frequencies. The results are in agreement with other examples of similar behavior of financial returns, provided in the relevant literature (see, e.g., Campbell et al., 1997, ch. 1).

[1] Such dependence assumptions are not significantly restrictive. For example, they should exclude cases for which the returns are given by $R_{t+1}=q_{t+1}-R_t$, where q_{t+1} has finite moments of order $s>a$.

[2] Hereafter, the notation FIGARCH will correspond to the cases where the fractional order of integration is larger than zero. Specifically, we will consider IGARCH as a special case of FIGARCH.

[3] We present the median of the values for each statistic because we observe many similarities between the patterns observed in the asset returns under consideration. In particular, when we use annual returns, the values of the statistics approximate the values that correspond to the normal distribution.

Table 1
Median of the kurtosis coefficients and the Jarque–Bera and Anderson–Darling test statistics of twelve major assets returns calculated at daily, weekly, monthly, quarterly semiannual and annual frequencies (p-values in parentheses).

	Frequency					
Medians of estimated	Daily	Weekly	Monthly	Quarterly	Semiannual	Annual
Kurtosis	7.290	6.230	5.015	4.024	4.344	3.140
Jarque–Bera	6446.000 (0.000)	825.638 (0.000)	94.273 (0.000)	16.285 (0.013)	8.552 (0.020)	1.861 (0.395)
Anderson–Darling	64.543 (0.000)	9.944 (0.000)	2.305 (0.000)	0.764 (0.046)	0.908 (0.019)	0.377 (0.391)

Next we focus on the memory characteristics of the conditional variance of the returns. To this end we fit a FIGARCH $(1,d,1)$ model, described by

$$\begin{aligned}&R_t = c + \sigma_t\varepsilon_t, \varepsilon_t \sim NIID(0,1)\\&(1-\beta L)\sigma_t^2 = \omega + \left(1-\beta L-(1+\varphi L)(1-L)^d\right)(R_t-c)^2,\end{aligned} \tag{1}$$

to every daily returns' series, where L is the lag operator.[4] Following the established terminology, σ_t^2 will be referred to as the conditional variance of R_t.[5] In all cases, the order of fractional integration, d, appears to be significantly different from zero, ranging from 0.28 (hogs) to 0.47 (S&P 500). Moreover, the robust Wald test statistic in all cases rejects the null hypothesis of a GARCH (1,1) against the alternative of a FIGARCH $(1,d,1)$ model. The analytical results can be found in the supplemental file of the paper. Baillie et al. (2007) examined a subset of our data (specifically, cattle, corn, gasoline, gold, hogs and soybeans) for a different sample period and reported similar results, suggesting that long memory is not a spurious – sample dependent – feature of financial returns.

Overall, the combined evidence from our dataset suggests the simultaneous presence of long memory in the conditional variance (where infinite variance is a direct implication of the FIGARCH specification) and aggregational Gaussianity for all series under consideration. Another way to examine for the presence of infinite variance would be an accurate estimation of the tail index, α, of the returns' distribution. This approach, however, requires extremely large datasets in order to produce reliable results, because the estimators of α are often biased, either upwards (e.g., McCulloch, 1997; Weron, 2001) or downwards (McCulloch, 1997). Note that in cases where aggregational Gaussianity holds, the task becomes even more difficult, because the tested hypotheses are $\{\alpha=2\}$ against $\{\alpha>2\}$, the latter corresponding to finite variance (see also Jach and Kokoszka, 2010).

3. Aggregational Gaussianity under barely infinite variance

Let R_t be the one-period (say daily) return on an asset, defined as $R_t = p_t - p_{t-1}$, where p_t is the natural logarithm of the price of the particular asset. In a similar fashion we define the k-period (say weekly or monthly) return $R_\tau(k)$ as:

$$R_\tau(k) = p_{\tau k} - p_{(\tau-1)k} = \sum_{i=1}^{k} R_{(\tau-1)k+i}. \tag{2}$$

The new index, τ, is introduced for notational simplicity, representing the k-period interval in terms of t. The series of k-period non-overlapping returns will be of the form $\{\ldots, p_{t-k}-p_{t-2k}, p_t-p_{t-k}, \quad p_{t+k}-p_t, \ldots\}$. This means that one unit in terms of τ will correspond to k units in terms of t.

Next, let us assume that the one-period returns follow a FIGARCH (1,d,1) process as it is described in (1). We shall attempt to answer the following question: Given that R_t follows a process with infinite variance, how does the distribution of $R_\tau(k)$ behave as the returns horizon k increases? To answer this question, we must examine whether the probabilistic properties of R_t are such that enable the application of a relevant limit theorem. To this end, let us first briefly discuss the case where R_t is a covariance stationary GARCH process. This process is also $\beta-$ mixing with exponential decay (see Carrasco and Chen, 2002; Francq and Zakoian, 2006). Since a $\beta-$ mixing process is also $\alpha-$ mixing, we can appeal to the central limit theorem by Ibragimov (1962) and conclude that as $k \to \infty$, the sequence $R_\tau(k)$ converges in law to the normal distribution. A similar result in a different context was obtained by Diebold (1988) who showed that the GARCH effects tend to disappear under temporal aggregation.

[4] The FIGARCH parameters are estimated using the G@RCH module of OxMetrics 6 and the Whittle estimators are obtained with the Matlab codes provided by Katsumi Shimotsu (https://sites.google.com/site/katsumishimotsu/home/resources).

[5] This term will be used even when $Var(R_t) = \infty$. Note also that under the FIGARCH hypothesis the variance and the kurtosis of the returns are infinite. Therefore, we cannot claim that any convergence takes place for the test statistics in Table 1, as the sample size increases.

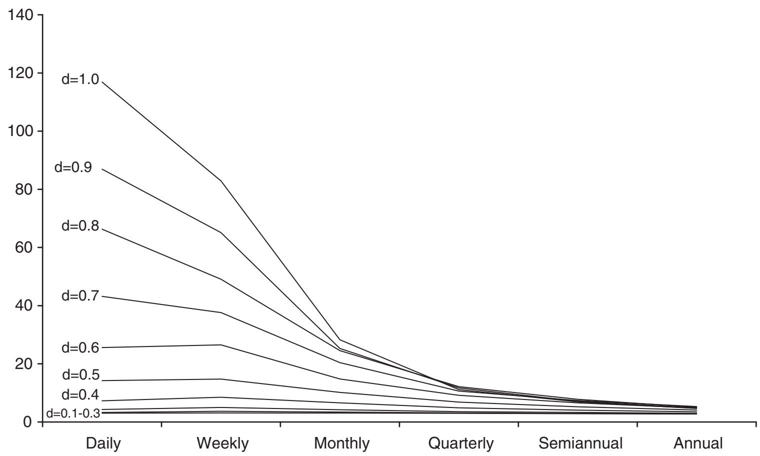

Fig. 1. Median kurtosis coefficients of 1000 simulated FIGARCH $(1,d,1)$ series across different frequencies with $\phi=0.1$, $\beta=0.2$ and $d\in\{0.1,0.2,\cdots,0.9,1\}$.

Let us now focus on the case that the returns are generated by a FIGARCH model. Douc et al. (2008) provide a proof for the stationarity of some FIGARCH $(0,d,0)$ models.[6] However, the variance of a well defined stationary FIGARCH process is infinite, hence Ibragimov's central limit theorem does not apply. Moreover, the results of Diebold (1988) are derived under the assumption of a covariance stationary GARCH process, which means that they do not cover any of the FIGARCH subcases (namely, the cases $0<d<1$, or the IGARCH, $d=1$). The presence of infinite variance seems to suggest that we must move away from the central limit theorem into limit theorems developed for the case of random variables with infinite variances. Historically, the problem described above was first dealt with by Lévy (1935) in the context of independent and identically distributed (*iid*) random variables, and later by Ibragimov and Linnik (1971) for the case of mixing random variables (see Kourogenis and Pittis, 2011, for an extensive discussion). Given the infinite variance of R_t, one could argue that it is reasonable to assume that for some $a<2$, all the moments of order $\delta>a$ of R_t are infinite. If this were the case, it would have implied that: (a) the limiting distribution of the appropriately standardized $R_\tau(k)$ is not normal; and (b) the standardizing sequence of $R_\tau(k)$ cannot be of the form $c\sqrt{k}$, where c is a positive constant.

However, the case of FIGARCH is different; a FIGARCH process exhibits barely infinite variance, meaning that $E|R_1|^\delta<\infty$ for every $\delta\in[0,2)$ (see Corollary 1 in Baillie et al., 1996; Douc et al., 2008; Kourogenis and Pittis, 2008, etc.). In this case, despite having infinite variance, the distribution of R_t belongs to the domain of non-normal attraction of the normal law (see Embrechts et al., 2003 p. 80). This implies that if $\{X_t\}$ is a sequence of iid random variables, each one having the distribution of R_t, there exists a sequence $\{g_k\}$, necessarily of the form $g_k=L(k)\sqrt{k}$, such that $\sum_{t=1}^{k}X_t/g_k$ weakly converges to the normal distribution. The function $L(k)$ is of particular interest. It is usually referred to as "slowly varying (at infinity)" meaning that $L(tx)/L(x)\to 1$ as $x\to\infty$ for every $t>0$. For dependent sequences, the limit theorems that ensure this result, are produced by Bradley (1988) and Peligrad (1990) for $\rho-$ mixing and $\phi-$ mixing sequences, respectively (see Kourogenis and Pittis, 2008, 2011). These results show that the finite variance assumption is not necessary for the central limit theorem. Specifically, for strictly stationary sequences, the central limit theorem requires that the truncated moment function, defined by $H(x)=ER_1^2 I_{|R_1|\le x}$, is slowly varying as $x\to\infty$. In fact, the condition of slow variation of $H(x)$ is both necessary and sufficient for the distribution of R_t to lie in the domain of attraction of the normal distribution (see Ibragimov and Linnik, 1971). The requirement that $H(x)$ is a slowly varying function is equivalent to the condition:

$$E|R_1|^\delta<\infty, 0\le\delta<2 \tag{3}$$

(see Bradley, 1988, p. 314). The latter condition amounts to saying that the R_t's have *just barely* infinite variance. This implies that the central limit theorem may hold even in cases where the variance of the R_t's is infinite, provided that all the moments of order $\delta<2$ are finite.

The preceding discussion suggests that the empirical features of aggregational Gaussianity and infinite variance of financial returns can coincide due to the limit theorems for mixing sequences with barely infinite variance. However, one word of caution is in order. The limit theorems of Bradley (1988) and Peligrad (1990) concern $\rho-$ mixing and $\phi-$ mixing sequences, respectively, while the mixing properties of FIGARCH processes are still unexplored. However, it is worth mentioning the results by Francq and Zakoian (2006) that an IGARCH process is $\beta-$ mixing with exponential decay, and by Zhang and Lin (2012) that aggregational Gaussianity can be directly derived for a general class of GARCH (including IGARCH) models with just barely infinite variance. Since there is no proof to date that convergence to normality under aggregation holds for a FIGARCH process, the

[6] To our knowledge, there is no explicit proof of the stationarity of the general FIGARCH(p,d,q) model. One way to provide such a proof would be to obtain parametric restrictions on the FIGARCH(p,d,q) model that satisfy the conditions of Douc et al. (2008). Such a task lies beyond the scope of this paper.

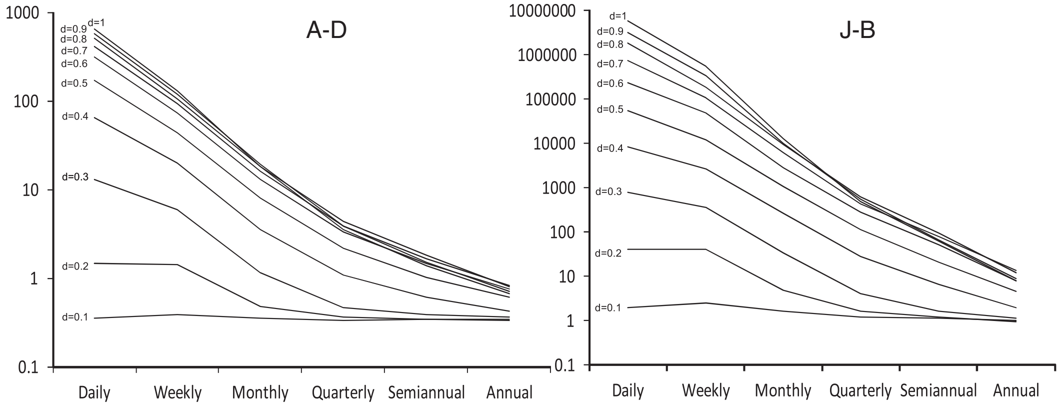

Fig. 2. Medians of Anderson–Darling (A–D) and Jarque–Bera (J–B) statistics of 1000 Simulated FIGARCH $(1,d,1)$ series across different frequencies with $\phi = 0.1$, $\beta = 0.2$ and $d \in \{0.1, 0.2, \cdots, 0.9, 1\}$. Vertical axes are in logarithmic scale.

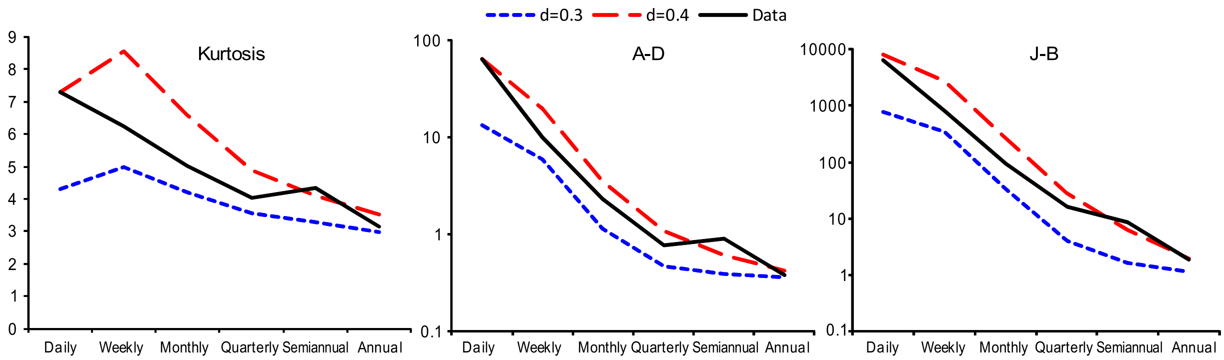

Fig. 3. Median kurtosis coefficients and medians of the values of Anderson–Darling (A–D) and Jarque–Bera (J–B) statistics of the asset returns, against the corresponding medians of 1000 simulated FIGARCH $(1,d,1)$ series across different frequencies with $\phi = 0.1$, $\beta = 0.2$ and $d \in \{0.3, 0.4\}$. The vertical axes for A–D and J–B are in logarithmic scale.

above-mentioned theorems should be used with caution. In the next section, however, we show that as the returns horizon increases, the degree of non-normality of a temporally aggregated FIGARCH process seems to decrease at a comparable rate with that of temporally aggregated IGARCH, while the latter is converging to the normal distribution (see Zhang and Lin, 2012, Theorem 2).

4. Identifying aggregational Gaussianity under FIGARCH

In the previous section we have clarified that aggregational Gaussianity is potentially compatible with the assumption that returns over the shortest horizon (say daily) follow a FIGARCH process. However, the empirical validation of this conjecture using formal statistical methods is not straightforward. The usual procedure for assessing whether an empirical distribution is normal, involves the use of a set of normality measures, such as the skewness, a_3, and kurtosis, a_4, coefficients and the Anderson–Darling (A–D) and Jarque–Bera (J–B) test statistics. To this end, establishing aggregational Gaussianity would imply to estimate these coefficients/statistics over various frequencies, and observe whether their values approach the values that correspond to the normal distribution as the frequency of observation (returns horizon) decreases (increases). Note that none of the aforementioned normality measures are expected to converge as the sample size increases, due to the infinite variance of FIGARCH returns. Therefore, we investigate convergence *only* in the frequency and *not* in the time domain.

In order to examine the behavior of the normality measures of $R_\tau(k)$ as k increases, we conduct a Monte Carlo experiment under the assumption that the one-period returns, R_t, are FIGARCH $(1,d,1)$. Concerning the conditional variance parameters of the FIGARCH $(1,d,1)$ model we set $(\phi,\beta) \in \{(0,0),(0.1,0.2),(0.5,0.5)\}$, while for the long memory parameter we set $d \in \{0.1,0.2,\ldots,0.9,1\}$ for the first two specifications, and $d \in \{0.1,0.2,\ldots,0.5\}$ for the case $(\phi,\beta) = (0.5,0.5)$, producing overall 25 different scenarios.[7] For each scenario we generate 1000 FIGARCH $(1,d,1)$ series of length equal to 10697, which is the number of daily observations for an asset for a period starting at 01/01/1970 and ending at 12/31/2010. For each of these 1000 replications, we generate five more

[7] When $(\phi,\beta) = (0.5,0.5)$, d is restricted so that the sufficient conditions in Baillie et al. (1996) for the positivity of conditional variances are not violated.

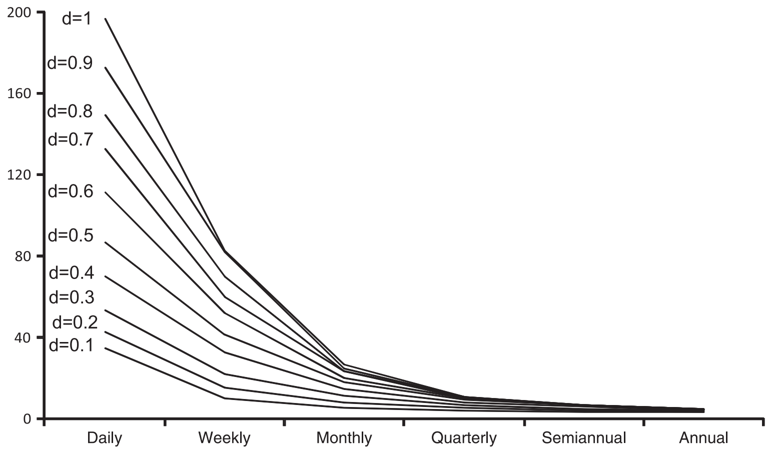

Fig. 4. Median kurtosis coefficients of 1000 simulated FIGARCH $(1,d,1)$ series across different frequencies with $\phi = 0.1$, $\beta = 0.2$, $d \in \{0.1, 0.2, \cdots, 0.9, 1\}$ and $\varepsilon_t \sim t(3)$.

series, $R_\tau(k)$, $k = 5$, 20, 60, 120, and 240, according to (2), corresponding to weekly, monthly, quarterly, semi-annual and annual frequencies. Note that the number of observations decreases with k; in particular we end up with 2140, 492, 164, 82 and 41 observations for $k = 5$, 20, 60, 120, and 240, respectively.

First, we estimate the kurtosis coefficient for all the available frequencies and we calculate the median across the 1000 replications for each frequency. Visual inspection of the results reveals exactly the same pattern in all three cases. Economizing on space, in Fig. 1 we present the results only for case $(\phi,\beta) = (0.1,0.2)$.

Exactly the same pattern is followed by the values of AD and JB, as it is illustrated in Fig. 2 (again for $(\phi,\beta) = (0.1,0.2)$).

Figs. 1 and 2 show that as the observation frequency decreases, the values of the three normality measures when $0.1 \le d \le 0.9$ decrease at a rate comparable to that of the corresponding values of the IGARCH (1,1) process (i.e. when $d = 1$), which converges to the normal distribution.

In Fig. 3 we compare the median of the estimated kurtosis coefficients, and the values of AD and JB of the asset returns, against the corresponding medians of the simulated FIGARCH series for $d \in \{0.3, 0.4\}$. It is worth noticing the similarity in their behavior.

The results may be summarized as follows:

(i) Both Monte Carlo and real data values exhibit a significant decrease as the returns horizon increases from one day to one year.
(ii) The values that correspond to the real data, appear to be bounded in almost all horizons by the Monte Carlo values for the cases $d \in \{0.3, 0.4\}$. It is worth noting that the median of the estimated fractional parameters, d, of the returns series under consideration is 0.406.

Next, we perform the same experiment as above, with $(\phi,\beta) = (0.1,0.2)$ and ε_ts in (1) being iid random variables that follow the student's t distribution with three degrees of freedom ($t(3)$). Note that in this case, the skewness of the ε_ts is not defined, while their kurtosis is infinite. Their finite variance, however, seems to suffice for the aggregate returns to exhibit similar behavior with the cases where ε_ts are normal. Fig. 4 illustrates the median kurtosis coefficients as the frequency decreases.

The kurtosis coefficients of the daily and weekly returns are considerably higher compared to the corresponding kurtosis coefficients when the ε_ts are normal. These differences, however, become negligible when the returns horizon becomes equal or larger than a month. The same pattern is observed for the AD and JB statistics, which we choose not to present for space economy.

5. Conclusions

Motivated by empirical evidence, we explain why there is no paradox in admitting the simultaneous existence of aggregational Gaussianity and infinite variance in financial returns. Our theoretical explanation derives from the probability theory literature and, in particular, from limit theorems for mixing processes with barely infinite variance.

The limit theorems by Bradley (1988) and Peligrad (1990) for $\rho-$ mixing and $\phi-$ mixing sequences with barely infinite variance, respectively, ensure the coincidence of the empirical features of aggregational Gaussianity and infinite variance. On the other hand, we point out that to date, there are no results that classify a FIGARCH process as $\rho-$ mixing or $\phi-$ mixing. Nevertheless, by means of a Monte Carlo experiment, we show that the temporal aggregation of a FIGARCH $(1,d,1)$ process results in random variables that approximate normality.

In order to provide some additional empirical evidence supporting the coexistence of FIGARCH effects in high frequency data and aggregational Gaussianity, we use a dataset with twelve major financial assets. We examine whether the rate at which the empirical distributions converge to normality can be explained by a FIGARCH specification. We show that a set of normality measures estimated from our dataset follow the same pattern as the corresponding normality measures when estimated from a simulated FIGARCH $(1,d,1)$ process, with d estimated from the dataset.

The results of this paper are both of theoretical and practical merit. First, they support the use of knife-edge infinite variance stationary processes, such as FIGARCH, for the specification of financial returns, without being in conflict with the empirical evidence suggesting that long horizon returns are almost normal. Second, they provide empirical evidence that supports the use of the normal distribution for the specification of long horizon returns, and at the same time, the use of fat tail distributions for the specification of returns at high frequencies.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jempfin.2012.11.003.

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